Newtonian Quantum Gravity

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Abstract

A Newtonian approach to quantum gravity is studied. At least for weak gravitational fields it should be a valid approximation. Such an approach could be used to point out problems and prospects inherent in a more exact theory of quantum gravity, yet to be discovered. Newtonian quantum gravity, e.g., shows promise for prohibiting black holes altogether (which would eliminate singularities and also solve the black hole information paradox), breaks the equivalence principle of general relativity, and supports non-local interactions (quantum entanglement). Its predictions should also be testable at length scales well above the "Planck scale", by high-precision experiments feasible even with existing technology. As an illustration of the theory, it turns out that the solar system, superficially, perfectly well can be described as a quantum gravitational system, provided that the l quantum number has its maximum value, $n-1$. This results exactly in Kepler's third law. If also the m quantum number has its maximum value ($\pm l$) the probability density has a very narrow torus-like form, centered around the classical planetary orbits. However, as the probability density is independent of the azimuthal angle ϕ there is, from quantum gravity arguments, no reason for planets to be located in any unique place along the orbit (or even *in* an orbit for $m \neq \pm l$). This is, in essence, a reflection of the "measurement problem" inherent in all quantum descriptions.

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The greatest fundamental challenge facing theoretical physics has for many years been to reconcile gravity with quantum physics. There have been numerous attempts to do so, but so far there is no established and experimentally/observationally tested theory of "quantum gravity", the two main contenders presently being string theory [1] and loop quantum gravity [2], with "outsiders" like twistor theory [3], non-commutative geometry [4], etc.

The motivations for studying newtonian quantum gravity are:

1) Quantum theory is supposed to be universal, i.e., it should be valid on all length scales and for all objects, as there in principle exists no size/charge/mass-limit to its applicability. In atomic physics the practical restriction comes about due to the fact that there is a limit to arbitrarily large atomic nuclei as, i) the Coulomb force between protons is repulsive, eventually overpowering the strong nuclear force trying to hold the nucleus together, ii) the additional weak force makes neutron-rich nuclei decay before they grow too large. Also, the electric charge comes in both positive and negative, and as a result a big lump of matter is almost always electrically neutral[1]. Neither of these effects are present in "pure" quantum gravity.

2) For weak gravitational fields the newtonian theory should be sufficient. The weak-field newtonian limit is even used for determining the constant κ in Einstein's field equations of general relativity $G_{\mu\nu} = \kappa T_{\mu\nu}$. The newtonian limit is also almost always sufficient for practical purposes in non-quantum gravity, except for a handful of extreme cases (notably black holes and the very early universe), although high-precision experiments in e.g. the solar system can and do show deviations from the newtonian theory, always in favor of general relativity [5].

3) Even for strong gravitational fields the newtonian picture gives the same prediction as general relativity for the Schwarzschild radius of a spherically symmetric, non-rotating black hole, and correct order of magnitude results for neutron stars and cosmology. This could make it possible to deduce at least qualitative results about strongly coupled quantum gravity, as the newtonian viewpoint should give reliable first order quantum gravitational results.

On the other hand would any "absurd" results obtained from newtonian quantum gravity, deviating from observations, implicate either that:

[1] The same also applies for e.g. the strong force, as the three different color charges ("red", "green", "blue") always combine to produce color-neutral hadrons and bulk matter.

A) General relativity cannot be quantized[2]. An unsuccessful special case (the weak field limit) would disprove the general case, whereas the opposite is not true. (A vindicated weak field limit will not prove that the general theory is also correct.)

or

B) Quantum mechanics fails at "macroscopic" distances and for macroscopic objects. This would mean that we in gravity have a unique opportunity to understand the "measurement problem" in quantum mechanics, as proposed by e.g. Károlyházy [7] and Penrose [8]. In that case we can use gravity to probe the transition between quantum $\rightarrow$ classical behavior in detail, i.e. get experimental facts on where, how and when the inherently undecided, subjective quantum world of superpositions turns into the familiar objective classical everyday world around us. One could, at least in principle, envisage a test carried out in a free-falling (e.g. satellite) environment where one alters m (the gravitational "test-charge") and M (the gravitational "source-charge") until the expected quantum gravity results are observed, to obtain a limit of where the quantum mechanical treatment breaks down, hance making an experimental determination of the border between "quantum" and "classical", i.e. solving the quantum mechanical measurement problem.

In newtonian quantum gravity, at least as long as the system can be approximately treated as a 2-body problem, it is possible to use the mathematical identity between the electrostatic Coulomb force in the hydrogen atom, and Newton's static gravitational force under the substitution $e^2/4\pi\epsilon_0 \rightarrow GmM$. Therefore all analytical results from elementary quantum physics directly lifts over to the quantum gravity case. For weak electromagnetic fields, as in the hydrogen atom, the electrodynamic corrections to the static Coulomb field are very small, making the approximation excellent. The same applies to quantum gravity, dynamical effects from general relativity are negligible to a very high degree for weak gravitational fields. A gravitationally bound 2-body system should then exhibit exactly the same type of "spectrum" as a hydrogen atom, but emitted in (unobservable) graviton form instead of photons (easily detectable as atomic spectra already in the 19th century).

For a free-falling 2-body system, e.g. in a satellite experiment enclosed in

[2]This is an automatic consequence of "emergent" gravity, e.g. Sakharov's theory [6], where gravity is a non-fundamental interaction and rather a macroscopic consequence of other forces and fields.

a spherical vessel, it should in principle be possible to measure the excitation energies for a suitable system. An analogous result has seemingly already been accomplished for neutrons in the gravitational field of the earth [9], although there are some quantum gravity ambiguities as noted below.

For hydrogen-like (one electron) atoms, in the dominant Coulomb central-field approximation, the energy levels depend only on the principal quantum number, $n = 1, 2, 3, ...$

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2}\frac{Z^2}{n^2} = -E_H\frac{Z^2}{n^2}, \tag{1}$$

where $E_H \simeq 13.6$ eV is the ionization energy, i.e. the energy required to free the electron from the proton, and Z the number of protons in the nucleus.

The Bohr-radius, a_0, the innermost radius of circular orbits in the old semi-classical Bohr-model, and also the distance r for which the probability density of the Schrödinger equation for the Hydrogen ground-state peaks, is

$$a_0 = \frac{h^2\epsilon_0}{\pi me^2}, \tag{2}$$

whereas the expectation value for the electron-nucleus separation is

$$\langle r\rangle_{hydrogen} \simeq \frac{n^2}{Z}a_0 = \frac{n^2h^2\epsilon_0}{Z\pi me^2}. \tag{3}$$

A comparison between the Coulomb potential in Hydrogen-like atoms

$$V_{hydrogen} = -\frac{Ze^2}{4\pi\epsilon_0 r}, \tag{4}$$

and the Newtonian gravitational potential between two masses m and M

$$V_{grav} = -\frac{GmM}{r}, \tag{5}$$

allows us to obtain all results of the gravitational case by the simple substitution

$$\frac{Ze^2}{4\pi\epsilon_0} \to GmM, \tag{6}$$

in the well-known formulas for the Hydrogen atom.

For instance, the gravitational "Bohr-radius", b_0, becomes

$$b_0 = \frac{h^2}{4\pi^2 Gm^2M} = \frac{\hbar^2}{Gm^2M}, \tag{7}$$

and the quantum-gravitational energy levels are

$$E_n(grav) = -\frac{2\pi^2 G^2 m^3 M^2}{h^2}\frac{1}{n^2} = -\frac{G^2 m^3 M^2}{2\hbar^2}\frac{1}{n^2} = -E_g\frac{1}{n^2}, \tag{8}$$

here again $E_g = G^2 m^3 M^2/2\hbar^2$ is the energy required to totally free the mass m from M in analogy to the Hydrogen case, whereas the expectation value for the separation is

$$\langle r\rangle_{grav} \simeq n^2 b_0 = \frac{n^2\hbar^2}{Gm^2 M}. \tag{9}$$

Also all the analytical solutions to the Schrödinger equation, the hydrogen wave-functions, carry over to the gravitational case with the simple substitution $a_0 \to b_0$.

$$\psi_{nlm} = R(r)\Theta(\theta)\Phi(\phi) = N_{nlm}R_{nl}Y_{lm}, \tag{10}$$

where N_{nlm} is the normalization constant, R_{nl} the radial wavefunction, and Y_{lm}, the spherical harmonics, contain the angular part of the wavefunction.

Let us examine some concrete cases to obtain a feeling for these relations: For a two-body problem composed of proton and electron $b_0 \simeq 10^{29}$ m, several orders of magnitude larger than the size of the observable universe ($\simeq 10^{26}$ m), whereas $E_g \simeq 10^{-78}$ eV. For two neutrons $b_0 \simeq 10^{22}$ m, $E_g \simeq 10^{-68}$ eV. For the earth and sun (approximated as a two-body problem for illustrative reasons) one gets $b_0 \simeq 10^{-138}$ m, an absurdly small ground state separation, and $E_g \simeq 10^{182}$ J, which is unphysical as the binding energy $E_g \gg mc^2 \simeq 10^{42}$ J. We will see below how to deal with these "unphysical" cases and how the physical picture somewhat surprisingly is connected to the Schwarzschild radius. For a better 2-body application, let us consider a binary neutron star system (one solar mass each), $b_0 \simeq 10^{-148}$ m, $E_g \simeq 10^{198}$ J $\gg mc^2 \simeq 10^{47}$ J, again unphysical. One could also ask how much m would have to be in a gravitational binary system (taking $m = M$) in order for b_0 to be, for example, one meter: $m \simeq 10^{-19} kg$, or the mass of a small virus. For a pair of "Planck-objects" $m = M \simeq 10^{-8}$ kg, we get, maybe not surprisingly, $b_0 \simeq 10^{-35}$ m (the "Planck length") and $E_g \simeq 10^9$ J (the "Planck energy") which also happens to be equal to mc^2. We could also ask for the binary system mass (again taking $m = M$) giving exactly the same numerical energy spectrum as for the Hydrogen atom, i.e. taking $E_g = E_H = 13.6$ eV, resulting in $m \simeq 10^{-13}$ kg, the mass of one human cell, and $b_0 \simeq 10^{-19}$ m.

One could ask for the mass, m, required to produce exactly the quantum gravitational energy spectrum of hydrogen in a gravitational field like that of earth, $M = M_\oplus \simeq 6 \times 10^{24}$ kg. This turns out to be $m \simeq 10^{-38}$ kg, or an equivalent mass-energy of $\sim 10^{-3}$ eV, comparable to the conjectured mass of neutrinos. As $b_0 \sim 1\mu$m in this case, only very highly excited states would be possible above the earth surface. The matter would of course be quite different around cosmic compact objects, for example the conjectured "preon stars" with masses comparable to the earth's and radii $\sim b_0$ [10].

We notice (e.g. through b_0) that the planets in the solar system must be in very highly excited quantum gravitational states. In that sense they are analogous to electrons in "Rydberg atoms" in atomic physics [11]. To obtain a good two-body approximation, let us study the sun-Jupiter system in a little more detail.

For excited states with $l \neq 0$, and very large n and l, the expectation value of the distance is

$$\langle r \rangle \simeq \frac{1}{2}(3n^2 - l^2) b_0, \tag{11}$$

however as that is for an ensemble (average over many measurements), for a single state it is in principle more appropriate to use the most probable radial distance ("radius" of orbital)

$$\tilde{r} = n^2 b_0, \tag{12}$$

as a measure for the expected separation. However, for n large and $l = l_{max} = n - 1$ the two coincide so that $\langle r \rangle = \tilde{r}$

The angular momentum for Jupiter around the sun is $L \simeq 2 \times 10^{43}$ Js, giving an l-quantum number of $l = L/\hbar \simeq 2 \times 10^{77}$. The most probable sun-Jupiter distance is given by $\tilde{r} = n^2 b_0 \geq l^2 b_0 \simeq 7.6 \times 10^{11}$ m, which is the same as the actual separation. $E_n = -E_g/n^2 \simeq -1.6 \times 10^{35}$ J, so the magnitude of the binding energy is much less than $mc^2 \simeq 1.8 \times 10^{44}$ J, making it physically allowed, and also of the same order of magnitude as its classical counterpart $-GmM/r \simeq -3.4 \times 10^{35}$ J. The sun-Jupiter system can thus seemingly be treated as a quantum gravitational 2-body system, provided that it is taken to have its maximally allowed value for its angular momentum ($l \simeq n$).

In fact, it is easy to show that for Kepler's law to apply, l must be very close to n:

The period of revolution can be written

$$T = \frac{2\pi m \tilde{r}^2}{L} = \frac{2\pi m \tilde{r}^2}{l\hbar}, \tag{13}$$

and assuming maximality for the angular momentum, $l \simeq n$, gives

$$T \simeq \frac{2\pi m\tilde{r}^2}{n\hbar}. \tag{14}$$

Solving the most probable distance, Eq. (12), for n gives

$$n = \frac{m\sqrt{GM\tilde{r}}}{\hbar}, \tag{15}$$

so that

$$T \simeq \frac{2\pi\tilde{r}^{3/2}}{\sqrt{GM}}, \tag{16}$$

which exactly is Kepler's law. So, the conclusion is that all the planets in the solar system are in maximally allowed angular momentum states quantum mechanically. The $l \simeq n$ quantum numbers are as follows: $l_{sun} \simeq 2 \times 10^{75}$, $l_{mercury} \simeq 8 \times 10^{72}$, $l_{venus} \simeq 2 \times 10^{74}$, $l_{earth} \simeq 3 \times 10^{74}$, $l_{mars} \simeq 4 \times 10^{73}$, $l_{jupiter} \simeq 2 \times 10^{77}$, $l_{saturn} \simeq 8 \times 10^{76}$, $l_{uranus} \simeq 2 \times 10^{76}$, $l_{neptune} \simeq 2 \times 10^{76}$, $l_{pluto} \simeq 3 \times 10^{72}$. Even though the maximality of L and L_z are automatic from the classical description, it is far from obvious why the same should result from the more fundamental quantum treatment, as noted below.

For states with $l = l_{max} = n - 1$ and $m = \pm l$: i) There is only one peak, at $r = \tilde{r}$, for the radial probability density, and the "spread" (variance) in the r-direction is given by[3] $\Delta r = \sqrt{\langle r^2\rangle - \langle r\rangle^2} \simeq n^{3/2} b_0/2$. For the earth-sun system it means $\Delta r \simeq 10^{-26}$ m, ii) The angular θ-part of the wavefunction for maximal m-quantum number $|m| = l$, is $\propto \sin^l\theta$. The probability density thus goes as $sin^{2l}\theta$ in the θ-direction, meaning that only $\theta = \pi/2$ is nonvanishing for large l. The azimuthal (ϕ) part of the angular wavefunction Y_{lm} is purely imaginary, making it drop out of the probability density, so that *all* values of ϕ are equally likely. (This ϕ-symmetry is a consequence of conservation of angular momentum in a central potential.) The total planetary probability density is thus "doughnut" (torus-like) shaped, narrowly peaking around the classical trajectory.

So, at first sight, it seems like the solar system is perfectly described as a quantum gravitational system. It even seems reasonable. Gravity totally dominates as all other forces, especially the only other known force with infinite reach, the electromagnetic, cancel due to charge neutrality. The

[3] The hydrogen wavefunctions for the gravitational case give $\langle r^2\rangle = [5n^2 + 1 - 3l(l+1)]n^2 b_0^2/2$ and $\langle r\rangle = [3n^2 - l(l+1)]b_0/2$.

solar system could thus be seen as a test-vehicle for quantum gravity. In the solar system the sun totally dominates the gravitational field, making the central field approximation an excellent one, even though it in principle is an N-body problem. Contrast this to the case of multielectron atoms in atomic physics where all electrons carry the same charge (1/N of the charge of the nucleus), making the central field approximation a very bad one.

However, from a quantum gravity standpoint, the system could be in any and all of the degenerate states, and usually at the same time, so typical for quantum mechanical superposition. Even for given energy and angular momentum there is no reason for the planets to be in any particular eigenstate at all of the $2l+1$ allowed, and certainly not exclusively $m = \pm l$. The radial probability distribution in general has $n - l$ maxima. Thus, only for $l = l_{max} = n - 1$ has it got a unique, highly peaked maximum. The degeneracy for a given n is n^2. Whenever $l < l_{max}$, the radial wavefunction is highly oscillatory in r as it has $n - l$ nodes. The same goes for the angular distribution as there in general are $l - m$ nodes in the θ-direction. For a general $R_{nl}Y_{lm}$ the planets could be "all over the place", and if this weren't bad enough, according to quantum mechanics the solar system more probable than not should be in simultaneous, co-existing superposed states with different quantum numbers as is generic in atomic physics. Consequently, newtonian quantum gravity cannot solve the quantum mechanical measurement problem, perhaps because it lacks the non-linear terms conjectured to be needed [8].

To get the innermost allowed physical orbit for any "test-particle", m, we must impose the physical restriction that the binding energy cannot exceed the test particle energy, thus

$$E_g(max) = mc^2. \tag{17}$$

As E_g can be written

$$E_g = \frac{GmM}{2b_0}, \tag{18}$$

we get

$$b_0(min) = \frac{GM}{2c^2} = \frac{R_S}{4}, \tag{19}$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius. The expression $b_0(min)$ gives a limit for b_0 of the system to be physically attainable. It is amusing to see how close $b_0(min)$ is to R_S and one cannot help speculate that a more complete theory of quantum gravity could ensure that $r > R_S$ always, and thus forbid black holes altogether[4]. The object M must be put together somehow, but if $r_{min} > R_S$ it can never accrete enough matter to become a black hole, as the infalling mass (energy) instead will be radiated away in its totality (in gravitons), making a black hole state impossible[5] [12]. This would, in an unexpected way, resolve the black hole information loss paradox. Even though $r = R_S$ represents no real singularity, as it can be removed by a coordinate transformation, anything moving inside $r < R_S$ will, according to classical general relativity, in a (short) finite proper time reach the true singularity at $r = 0$. If quantum gravity could ensure that $r > R_S$ always, gravity would of course be singularity free.

Let us also briefly look at radiative transitions. From the dipole approximation in atomic physics an elementary quantum (photon) transition requires $\Delta l = \pm 1$. A quadrupole (graviton) approximation in quantum gravity instead requires $\Delta l = \pm 2$. So, a typical elementary energy transfer in a highly excited, gravitationally bound 2-body quantum gravitational system is

$$\Delta E = -E_g(n^{-2} - (n-2)^{-2}) \simeq \frac{4E_g}{n^3}. \tag{20}$$

For the earth-sun system this means $\Delta E \simeq 2 \times 10^{-20}$ eV, carried by a graviton with frequency $\nu \simeq 5 \times 10^{-6}$ Hz, and wavelength $\lambda \simeq 6 \times 10^{13}$ m $\simeq 400$ AU (1 AU being the mean distance between the earth and sun).

The average time required for each elementary quantum gravity transition to take place can be estimated roughly by $\Delta t \sim \hbar/\Delta E \simeq 3 \times 10^4$ s $\simeq$ 8h 20min. Thus the power radiated by a spontaneously emitted individual graviton is very roughly $\sim 10^{-43}$ W, compared to the prediction from the usual quadrupole formula (first non-vanishing contribution) in classical general relativity of $\simeq 300$ W for the total power. We also see that the gravitational force is not really conservative, even in the static newtonian approximation, but the difference is exceedingly small in the sun-earth system. The changes in kinetic and potential energies do not exactly balance, $\Delta K \neq \Delta U$, the difference being carried away by gravitons in steps of $\Delta l = 2$. Also, in quantum gravity there is gravitational radiation even in the spher-

[4]For the hydrogen atom the corresponding value is $a_0(min) \simeq 1.4 \times 10^{-15}$ m, or one-half the "classical electron radius", whereas $R_S \simeq 10^{-53}$ m, so that $a_0(min) \gg R_S$. But we implicitly already knew that. The Coulomb force does not turn atoms into black holes.

[5]For the classical case, the relation is even closer, $GmM/r_{min} = mc^2$, giving $r_{min} = R_S/2$, but then one cannot really speak of energy being carried away by gravitons.

ically symmetric case, which is forbidden according to the classical general relativistic description.

Let us now return to the experiment with neutrons in the gravitational field of the earth [9], claiming to have seen, for the first time, quantum gravitational states in the potential well formed by the approximately linear gravitational potential near the earth surface and a horizontal neutron mirror. An adjustable vertical gap between the mirror and a parallel neutron absorber above was found to be non-transparent for traversing neutrons for separations less than $\sim 15\mu$m (essentially due to the fact that the neutron ground state wavefunction then overlaps the absorber). As the neutron in such a well, from solving the Schrödinger equation, has a ground state wavefunction peaking at $\sim 10\mu$m, with a corresponding energy of $\simeq 1.4 \times 10^{-12}$ eV, the experimental result is interpreted to implicitly having verified, for the first time, a gravitational quantum state.

If we instead analyze the experiment in the framework of the present article, the same experimental setup gives $b_0 \simeq 9.5 \times 10^{-30}$ m, $E_g \simeq 2.2 \times 10^{35}$ eV. Close to the earth's surface, $\tilde{r} \simeq R_\oplus \simeq 6.4 \times 10^6$ m, the radius of the earth, giving $n \simeq 8.2 \times 10^{17}$, resulting in a typical energy for an elementary quantum gravity transition $\Delta E \simeq 4E_g/n^3 \simeq 1.6 \times 10^{-18}$ eV. For a cavity of $\Delta\tilde{r} = 15\mu$m, and $n \gg \Delta n \gg 1$, one gets $\Delta E = E_g b_0 \Delta\tilde{r}/\tilde{r}^2 \simeq 0.7 \times 10^{-12}$ eV $= 0.7$ peV, to be compared to the value 1.4 peV as quoted in [9]. Even though the present treatment gives a similar value for the required energy, it need *not* be the result of a *single* quantum gravity state as calculated in [9], but rather $\leq 10^6$ gravitons can be emitted/absorbed. From the treatment in this article it is thus not self-evident to see why the experimental apparatus [9] should be non-transparent to neutrons for vertical separations $\Delta h < 15\mu$m.

To appreciate the potential importance of this, let us digress briefly on the equivalence principle, the main conceptual pillar of general relativity. A classic example for illustrating it involves two rockets: One rocket stands firmly on the surface of the earth, while the other accelerates constantly in empty space with $a = g$. According to the equivalence principle there is no way to, locally, distinguish one from the other if one is not allowed/able to make outside observations, meaning that acceleration and gravity are equivalent. However, in the quantum gravity case, for example considering a neutron inside each rocket, there certainly *is* a difference: For the rocket standing on the earth the potential in the Schrödinger equation is $V = -GmM/r$, resulting in normal quantization as elaborated above. For the rocket accelerating in space, however, $V = 0$ and the energy levels are non-quantized (the neutron being a free particle until it hits the floor of the rocket). So the conclusion is that newtonian quantum gravity breaks the equivalence principle. Furthermore, to understand why a free-falling object classically accelerates radially downward in, e.g., the earth's gravitational field, the gravitons must, by conservation of momentum, be emitted in the direction opposite to the acceleration (at least the probability for emission must peak in that direction). Also, the quantum states with given n, l, m are in principle inherently stable, an outside perturbation being needed for the transition rate to be different from zero, just like in atomic physics. For macroscopic bodies this poses no problem as there in that case are abundant backgrounds of both gravitational and non-gravitational disturbances. For an elementary quantum gravity interaction, however, this problem seems much more severe, as the notion of free-fall loses its meaning as the quantum states become practically stable to spontaneous graviton emission. In fact, a bound quantum gravitational object does not fall at all as it is described by a stationary wavefunction, or a superposition of such.

Thus, the difference regarding quantized energy levels for an experiment with neutrons "falling" under the influence of earth's gravity *with* mirror (as in [9]) or *without* (above) shows that newtonian quantum gravity is dependent on global boundary conditions, where the boundary in principle can lie arbitrarily far away. This comes as no surprise, as the Schrödinger equation models the gravitational interaction as instantaneous, contrasted with the case in general relativity where the behavior in free-fall only depends on the local properties of mass-energy and the resulting spacetime curvature (out of which the mirror is not part due to its inherently non-gravitational interaction with the neutron) and causal connection as the gravitational interaction propagates with the speed of light. However, as several experiments on entangled quantum states, starting with Clauser/Freedman [13] and Aspect et al. [14], seem to be compatible with a non-local connection between quantum objects [15], this property of the Schrödinger equation does not, at least for the moment, seem to be a serious drawback for a theory of quantum gravity. One could even envisage a "delayed choice" experiment á la Wheeler, where the mirror is removed/inserted before the neutron reaches its position, meaning that we could alter the energy of the gravitons *after* they have been emitted.

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